Oppgave 2.1:

a) $f(x) = 2x^3 - x^2 - 4x - 5$

$$f'(x) = 2\cdot 3x^2 - 2x - 4 - 0 = \underline{\underline{6x^2 - 2x - 4}}$$

b) $f(x) = x^2\left(x^2 + x - 3\right) = x^4 + x^3 - 3x^2$

$$f'(x) = 4x^3 + 3x^2 - 3\cdot 2x = \underline{\underline{4x^3 + 3x^2 - 6x}}$$

c) $f(x) = \dfrac{2x^2 - 1}{x+3}$

$$f'(x) = \frac{(2\cdot 2x - 0)(x+3) - \left(2x^2 - 1\right)\cdot 1}{(x+3)^2} = \frac{4x(x+3) - \left(2x^2-1\right)}{(x+3)^2}$$

$$= \frac{4x^2 + 12x - 2x^2 + 1}{(x+3)^2} = \underline{\underline{\frac{2x^2 + 12x + 1}{(x+3)^2}}}$$

d) $f(x) = \dfrac{x^3 + 2x^2 - 1}{x^2 + 1}$

$$f'(x) = \frac{\left(3x^2 + 2\cdot 2x - 0\right)\left(x^2+1\right) - \left(x^3 + 2x^2 - 1\right)(2x + 0)}{\left(x^2+1\right)^2}$$

$$= \frac{3x^4 + 3x^2 + 4x^3 + 4x - 2x^4 - 4x^3 + 2x}{\left(x^2+1\right)^2} = \underline{\underline{\frac{x^4 + 3x^2 + 6x}{\left(x^2+1\right)^2}}}$$

Oppgave 2.2:

a) $f(x) = \sqrt{x^3} = \left(x^3\right)^{\frac{1}{2}} = x^{\frac{3}{2}}$

$$f'(x) = \tfrac{3}{2}x^{\frac{3}{2}-1} = \tfrac{3}{2}x^{\frac{1}{2}} = \underline{\underline{\tfrac{3}{2}\sqrt{x}}}$$

b) $f(x) = \dfrac{x-1}{\sqrt{x}} = \dfrac{x}{\sqrt{x}} - \dfrac{1}{\sqrt{x}} = \sqrt{x} - \left(\sqrt{x}\right)^{-1} = x^{\frac{1}{2}} - \left(x^{\frac{1}{2}}\right)^{-1} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

$$f'(x) = \tfrac{1}{2}x^{\frac{1}{2}-1} - \left(-\tfrac{1}{2}x^{-\frac{1}{2}-1}\right) = \tfrac{1}{2}x^{-\frac{1}{2}} + \tfrac{1}{2}x^{-\frac{3}{2}} = \tfrac{1}{2}\left(\frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^{\frac{3}{2}}}\right) = \tfrac{1}{2}\left(\frac{1}{\sqrt{x}}\cdot\frac{x}{x} + \frac{1}{x\sqrt{x}}\right)$$

$$= \underline{\underline{\frac{1}{2x\sqrt{x}}(x+1)}}$$

Vi kan også bruke derivasjonsregelen for en brøk, der vi på forhånd har funnet ut at $\dfrac{d}{dx}\left(\sqrt{x}\right) = \dfrac{1}{2\sqrt{x}}$ (se Eksempel 2.2a). Da får vi:

$$f'(x)=\frac{(1-0)\cdot\sqrt{x}-(x-1)\cdot\frac{1}{2\sqrt{x}}}{\left(\sqrt{x}\right)^2}=\frac{\sqrt{x}-\frac{x}{2\sqrt{x}}+\frac{1}{2\sqrt{x}}}{x}\cdot\frac{\sqrt{x}}{\sqrt{x}}=\frac{x-\frac{x}{2}+\frac{1}{2}}{x\sqrt{x}}=\frac{\frac{x}{2}+\frac{1}{2}}{x\sqrt{x}}$$

$$=\underline{\underline{\frac{x+1}{2x\sqrt{x}}}}$$

c) $$f(x)=\left(2\sqrt{x}-1\right)^2=\left(2\sqrt{x}\right)^2-2\cdot 2\sqrt{x}+1=4x-4x^{\frac{1}{2}}+1$$

$$f'(x)=4\cdot 1-4\cdot\tfrac{1}{2}x^{\frac{1}{2}-1}+0=4-2x^{-\frac{1}{2}}=4-2\cdot\frac{1}{x^{\frac{1}{2}}}=\underline{\underline{4-\frac{2}{\sqrt{x}}}}.$$

Oppgave 2.3:

a) $$f(x)=x^2\cdot\cos x$$

$$f'(x)=2x\cdot\cos x+x^2\cdot(-\sin x)=\underline{\underline{2x\cos x-x^2\sin x}}$$

b) $$f(x)=\frac{\cos x}{x}$$

$$f'(x)=\frac{(-\sin x)\cdot x-(\cos x)\cdot 1}{x^2}=\underline{\underline{\frac{-x\cdot\sin x-\cos x}{x^2}}}$$

c) $$f(x)=\sqrt{x}\cdot\tan x$$

$$f'(x)=\frac{1}{2\sqrt{x}}\cdot\tan x+\sqrt{x}\cdot\frac{1}{\cos^2 x}=\frac{1}{2\sqrt{x}}\cdot\frac{\sin x}{\cos x}+\frac{\sqrt{x}}{\cos^2 x}$$

$$=\frac{\sin x\cdot\cos x}{2\sqrt{x}\cdot\cos^2 x}+\frac{2\sqrt{x}\cdot\sqrt{x}}{2\sqrt{x}\cdot\cos^2 x}=\underline{\underline{\frac{\sin x\cdot\cos x+2x}{2\sqrt{x}\cdot\cos^2 x}}}.$$

Oppgave 2.4:

a) $$f(x)=x^2\cdot e^x$$

$$f'(x)=2x\cdot e^x+x^2\cdot e^x=\underline{\underline{\left(2x+x^2\right)e^x}}$$

b) $$f(x)=(2x-3)\cdot\ln\left(x^2\right)=(2x-3)\cdot(2\ln x)=(4x-6)\cdot\ln x$$

$$f'(x)=(4\cdot 1+0)\cdot\ln x+(4x-6)\cdot\tfrac{1}{x}=\underline{\underline{4\ln x+4-\tfrac{6}{x}}}$$

c) $$f(x)=x\cdot\ln\left(\sqrt{x}\right)=x\cdot\ln\left(x^{\frac{1}{2}}\right)=x\cdot\left(\tfrac{1}{2}\ln x\right)=\tfrac{1}{2}(x\cdot\ln x)$$

$$f'(x)=\tfrac{1}{2}\left(1\cdot\ln x+x\cdot\tfrac{1}{x}\right)=\underline{\underline{\tfrac{1}{2}(\ln x+1)}}$$

Oppgave 3.1:

a) $f(x)=(2x-1)^3=u^3$ der $u=2x-1$.

$$f'(x) = 3u^2 \cdot u' = 3(2x-1)^2 \cdot (2\cdot 1 - 0) = \underline{\underline{6(2x-1)^2}}$$

b) $f(x) = \left(x^2 - x\right)^2 = u^2$ der $u = x^2 - x$.

$$f'(x) = 2u \cdot u' = \underline{\underline{2\left(x^2 - x\right)\cdot(2x-1)}}$$

c) $f(x) = \sqrt{x^2+4} = \sqrt{u} = u^{\frac{1}{2}}$ der $u = x^2 + 4$.

$$f'(x) = \tfrac{1}{2}u^{\frac{1}{2}-1} \cdot u' = \tfrac{1}{\not{2}}u^{-\frac{1}{2}} \cdot (\not{2}x + 0) = x \cdot \frac{1}{u^{\frac{1}{2}}} = \underline{\underline{\frac{x}{\sqrt{x^2+4}}}}$$

d) $f(x) = e^{-x^2} = e^u$ der $u = -x^2$.

$$f'(x) = e^u \cdot u' = e^{-x^2} \cdot (-2x) = \underline{\underline{-2x \cdot e^{-x^2}}}$$

Oppgave 3.2:

a) $f(x) = \sin(3x) \quad \Rightarrow \quad f'(x) = \underline{\underline{3\cos(3x)}}$

b) $f(x) = e^{-2x} \quad \Rightarrow \quad f'(x) = \underline{\underline{-2e^{-2x}}}$

c) $f(x) = e^{-x} \cdot \cos(2x)$

$$\Leftrightarrow \quad f'(x) = -e^{-x} \cdot \cos(2x) + e^{-x} \cdot \left(-2\sin(2x)\right) = \underline{\underline{-e^{-x}\left(\cos(2x) + 2\sin(2x)\right)}}$$

d) $f(x) = x \cdot \ln(4x) \quad \Leftrightarrow \quad f'(x) = 1 \cdot \ln(4x) + x \cdot \frac{1}{x} = \underline{\underline{\ln(4x) + 1}}$